The Emission Allowance Offset Credit Price Spread: a Partial Equilibrium Model

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Abstract

The Kyoto Protocol established targets for curbing greenhouse gas emissions in order to mitigate climate change, and it introduced two kinds of market-based mechanisms: the emission allowance market and the carbon offset market. We identify stylized features of the two mechanisms with a partial equilibrium model. Our work is the first to derive a closed form solution incorporating most policy instruments, such as abatement and offset usage, and delivery risks in offsets. We show that market shocks will impact one market directly and the other indirectly, generating unequal price responses that affects the spread between the two compliance instruments. We show how the price spread between allowances and offsets is affected by market conditions such as the offset import limit, abatement and offset cost, penalty rate, emission cap, and baseline emissions.

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1. Introduction

The Kyoto Protocol established targets for curbing greenhouse gas (GHG) emissions. Emission allowances are granted to annex 1 parties (developed European countries), and they can be used for compliance or sold if total emissions are below their target. Also, the Kyoto Protocol implemented the Clean Development Mechanism (CDM) so that non-Annex 1 parties can voluntarily reduce emissions, generate carbon offset credits, and sell them to Annex 1 parties.

The CDM generates offset credits known as Certified Emission Reductions (CERs) that come in two types. Primary CERs (pCERs) are contracted for forward delivery before the emission reductions are approved by the United Nations and all pCERs bear the risk of non-completion.[3] Once pCERs are sold to a broker, they are called secondary CERs (sCERs) and the broker must bear the delivery risk if the emission reductions have not yet been approved by the United Nations. The objective of the CDM is to stimulate investment in climate change mitigation that transfers technologies from developed countries to host countries with the hope of promoting sustainable development.

Figure 1 shows historical data for EUA, sCER, and some examples of pCER prices from 2008-2009 in the European Union market (WorldBank, 2009). Even though all three represent the right to emit one tonne of carbon, both CERs trade at a discount to the EUA. Nazif (2013)

[3] A CDM project must proceed through the following steps in the project cycle before it is approved to issue CERs: (1) project design, (2) national approval, (3) validation, (4) registration, (5) monitoring, (6) verification, and finally (7) CER issuance. The timeline of the project cycle varies according to each project, but it usually takes more than a year. After the CER issuance is approved by the United Nations Clean Development Mechanism Executive Board, the CERs (primary CERs, specifically) can be sold from a project participant of the CDM project to any buyer. Therefore, buyers are exposed to risk that CER delivery is delayed, or that the CERs never arrive. We model primary CERs to reflect the underlying risk of the CDM project. For instant, pCER were issued from the Cartagena Landfill Gas Capture and Usage Project in Colombia (funded by Germany); the project was registered on Dec. 2012, which eliminates the possibility of delivery risk.

empirically showed two factors contribute to the persistent price spread: 1) The sCER-pCER spread is primarily due to delivery risk in the pCERs, while 2) the EUA-sCER spread is primarily due to an import limit on offset credits. The objective of limiting imports is to prevent CERs from flooding the market and driving EUA prices to zero, thus eliminating the incentive to reduce emissions in Annex 1 counties.

[Insert Figure 1 about here]

The price spread between EUAs and CERs is important for both regulators and regulated firms to understand. For example, an annex 1 party can purchase sCERs and achieve lower compliance cost, since sCERs trade at a discount to EUAs. Further, if an annex 1 party is willing to bear delivery risk, they can profit from the spread between pCERs and EUAs by buying cheap pCERs and selling expensive EUAs, thereby reducing compliance cost in exchange for bearing the pCER risk. We build a partial equilibrium model between an emission allowance market and an offset credit market. From this point forward we use the generic terms *allowances* and *offset credits* rather than the specific terms EUA and CER. Further, the term offset credits refers to the more general *primary offset credits* discussed above. Results for *secondary offset credits* can be obtained by assuming zero delivery risk in the analysis below.

There have been several empirical studies on the price spread between allowances and offset credits. Mansanet-Bataller et al. (2011) found energy prices and climate variables to have explanatory power over the price spread. Chevallier (2011 a,b) estimated the time varying correlation between allowance and secondary offset credit prices to be within the range [0.01, 0.9] using a multivariate GARCH framework. Given the findings of Nazif, and the wide range in time-varying correlation found by Chevallier, there is a need to model how policy variables and fundamentals in the allowance and offset markets interact to influence the price spread. However, there are only a few analytic studies on the matter.

World Bank (2009) stated that risk-averse investors tend to buy guaranteed secondary offset credits rather than primary offsets, and heterogeneity in risk preferences determines the price spread between the two. Carmona and Fehr (2011) constructed an equilibrium model that investigates the joint price dynamics between the two markets. Their conceptual model was quite general and could be applied to a number of emission regulatory frameworks, but the generality of the model obscured insights specific to the allowance and offset markets. Barrieu and Fehr (2011) developed a tractable price equilibrium model based on no-arbitrage pricing of the allowance-offset price spread. However, since they utilize an equilibrium model, one cannot map how primitives of the problem, such as parameters in the abatement cost function, influence the spread. Therefore, it is instructive to construct a single period partial equilibrium model based on primitives of the annex 1 and non-annex 1 party's cost functions. Such a model lends itself to a clear description of how policy and cost function parameters affect the price spread between allowances and offsets.

Our partial equilibrium model is the first analytic work to derive a closed form solution that incorporates policy instruments and delivery risks in the offset market. Our paper departs from previous equilibrium models by allowing both abatement and offset credit purchases to be choice variables in the regulated party's compliance cost minimization problem, delivering a closed form solution of the spread as a function of primitives of the regulated and non-regulated parties' problem and policy parameters.

Therefore, our paper answers the question of how and why allowance and offset credit prices can move in opposite directions. Comparative statics on our model shows that increasing the import limit or increasing the initial endowment of allowances decreases the price of allowances and increases the price of offsets, because these policy parameters induce a substitution of offsets for allowances which narrows the price spread. Additionally, our model shows that increasing the non-compliance penalty or increasing emissions increase the price of both allowances and offsets since these parameters increase demand for both compliance instruments. Since the analytic model cannot inform the relative sizes of the price increases we calibrate our model to determine the effect on the price spread. The calibrated model shows that increasing the non-compliance penalty and increasing emissions causes allowance prices to increase more than offset credit prices, thus increasing the price spread in both cases. Additional analysis in the calibrated model shows that an increase in the cost of abatement increases both the price of emission allowances and offset credits, but offset credits to a lesser degree. Thus narrowing the price spread and making offset credits more attractive. While increasing the cost of the offset project has the opposite effect, increasing the price spread and making allowances relatively more attractive than offsets.

The paper proceeds as follows: In section 2, we model two representative parties (uncapped and capped), derive the market equilibrium, and perform comparative statics; In section 3 we calibrate the model based on the parameters of European carbon market; in section 4 we conduct a sensitivity analysis and provide a graphical explanation of the market mechanics; and a final section concludes.

2. Model

In this section, we derive the permit market equilibrium by assuming two representative parties. A risk neutral representative party in Annex-1 countries is subject to a carbon emission cap, and it achieves compliance by engaging in a cost minimizing mix of the following activities: abating emissions, using emission allowances, and purchasing carbon offset credits. On the other hand, a risk neutral representative party in the non-Annex 1 countries is not subject to the emission cap; however, they can administer carbon offset projects to produce offset credits. Although the non-Annex 1 party is not subject to the emission cap, they effectively abate emissions on behalf of the Annex 1 party in exchange for payment for the offset credits generated. The representative capped party in Annex 1 thereby exports emissions to the uncapped party at the expense of buying offset credits (Montero, 2000; Mason and Plantinga, 2011). A more detailed description of each party's problem follows.

2.1. The Uncapped Party's Problem

The uncapped party is a price taker in the offset credit market, whose price is denoted by P_{offset}; the variable q is the number of offset credits generated; and $C_{offset}(q)$ is the convex cost function of the offset project generating q credits. The uncapped party maximizes profits by choosing the optimal level of offset credits in equation (1).

(1) $$\underset{q}{Max} \;\; P_{offset} \cdot q - C_{offset}(q)$$

Taking the derivative of equation (1) with respect to q defines the profit maximizing condition for the uncapped party,

$$(2) \qquad 0 = P_{offset} - MC_{offset}(q),$$

which implies that the marginal benefit of selling offset credits, P_{offset}, equals the marginal cost of generating offset credits, $MC_{offset}(q) = \frac{\partial C_{offset}(q)}{\partial q}$.

2.2 The Capped Party's Problem

The capped party can comply by abating emissions, buying allowances, or importing offset credits. The amount of offset credits purchased from the uncapped party is denoted by q, but the actual amount of usable offset credits may be less because of delivery risk and import limitations.

As Cormier and Bellassen (2013) note, sometimes offset credits are not delivered due to various problems that can arise during the course of the project. For example, political instability in the country or financial uncertainty lead to a number of the projects defaulting before offsets are delivered. We define $\varepsilon \in [0,1]$ to be the default factor, which is uncertain and represents the proportion of contracted offset credits that are delivered; therefore, delivered offsets equal $q\varepsilon$. The more risky an offset project is, the smaller the expected default factor.

Additionally, delivered offset credits can only be used up to an import limit defined by $\theta \in [0,1]$. The import limit is a fraction of the original emission allowances, L, endowed to the annex 1 party. Currently, the EU-ETS sets the import limit at θ =13.4% of EUA allocations on average (Chevallier, 2011b; WorldBank, 2008, 2009). [4]

Equation (3) takes into account how both delivery risk and the import limit causes the number of offsets received to be smaller than the expected number of offsets purchased.

(3) $$Q(\, q|\varepsilon, L, \theta\,) = \min[q\varepsilon, L\theta]$$

The capped party minimizes total compliance costs, which consists of two parts: cost of compliance and an expected penalty for noncompliance. The compliance cost minimization problem is conducted based on expectations over the uncertain default factor and uncertain emissions, which are assumed to be independent.

(4) $$\underset{u,q}{\text{Min}}\, E_y E_\varepsilon \left\{ \underbrace{C(u,q)}_{Cost\ of\ Compliance} + \underbrace{B(u,Q)}_{Penalty\ for\ Noncompliance} \right\}$$

Equation (4) has two control variables, emission abatement, u, and offset credits, q. In reality, additional allowances can be purchased from other firms to come into compliance, but this is precluded in our model by assuming the market consists of a representative party in the Annex 1 countries. Since our objective is to explain the spread between allowances and offsets, we accepted this simplification to maintain tractability of the model. Also, banking or borrowing allowances is another instrument for compliance, which can change the total number of allowances available in a specific compliance period. We abstract from this detail so we can maintain a simpler single period model.

The cost function for compliance is defined in equation (5) below.

(5) $$C(u,q) = TAC(u) + P_{offset} \cdot q$$

[4] In the EU-ETS, the import limit varies across countries and industries. Offset credits must be used for compliance by the end of each year. There is evidence that the import limit has been binding in the EU-ETS because produced CERs have been larger than the demand by the import limit.

The compliance cost function, $C(u,q)$, is defined by total abatement cost, $TAC(u)$, and any offset credit expenditures, $P_{offset} \cdot q$. The first term, $TAC(u)$, is a convex function of abatement. The parameter P_{offset} is the per unit price of offset credits which are delivered from the uncapped party.

The penalty for noncompliance is defined in equation (6).

(6) $$B(u,q) = P_{penalty} \cdot max[y - L - u - Q\ ,\ 0]$$

where $P_{penalty}$ is the penalty rate per tonne of carbon emissions over the cap; y is emissions, which are uncertain. We assume the regulator gives an initial allocation of allowances, L, for free to the capped party.[5] Penalties are levied if the capped firm emits more than the sum of allowances, abatement, and delivered offset credits.

Substituting (5) and (6) into equation (3), the capped party's objective is,

(7) $$\underset{u,q}{\text{Min}}\ \underbrace{TAC(u) + P_{offset} \cdot q}_{Cost\ of\ Compliance} + E_y E_\varepsilon \underbrace{\left[P_{penalty} \cdot max[y - L - u - \min[q\varepsilon, L\theta]\ ,\ 0]\right]}_{Potential\ Penalty\ of\ Noncompliance}$$

To minimize compliance costs of the capped party, we take the derivatives of equation (7) with respect to the control variables, abatement u, and offset credit purchase q.

(8) $$0 = \frac{\partial TAC(u)}{\partial u} + \frac{\partial P_{penalty} \cdot EE\ max[y - L - u - \min[q\varepsilon, L\theta],\ 0]}{\partial u} = MAC(u^*) - MB(u^*|q)$$

(9) $$0 = P_{offset} + \frac{\partial P_{penalty} \cdot EE\ max[y - L - u - \min[q\varepsilon, L\theta],\ 0]}{\partial q} = P_{offset} - MB(q^*|u)$$

Equation (8) shows that the marginal abatement cost, $MAC(u) = \frac{\partial TAC(u)}{\partial u}$, should be equal to the marginal benefit of abatement due to avoiding a penalty,

$MB(u|q) = -\frac{\partial P_{penalty} \cdot EE\ max[y - L - u - \min[q\varepsilon, L\theta],\ 0]}{\partial u}$. Similarly, equation (9) shows that the

[5] During the Phase 2 (2008~2012) in the EU-ETS, only 4% of allowances are auctioned off. (WorldBank, 2012)

marginal cost of buying offset credits should equal the marginal benefit of using offset credits to avoid a penalty, $MB(q^*|u) = -\frac{\partial P_{penalty} \cdot EE\ max[y-L-u-\min[q\varepsilon, L\theta],\ 0]}{\partial q}$.

2.3. Equilibrium and Comparative Statics

By solving equations (8) and (9) simultaneously, we derive the optimal abatement and offset credit levels, (u^*, q^*). Further, the equilibrium price of emission allowances equals the marginal cost and benefit of abatement, and the equilibrium price of offset credits equals the marginal cost and benefit of offset credits; equation (10) and (11) show how the prices of allowances and offsets are defined.

(10) $P_{allowance} = MAC(u^*) = MB(u^*|q)$

(11) $P_{offset} = MC(q^*) = MB(q^*|u)$

We derive the comparative statics of this equilibrium in Appendix A. In table 1, the result reveals that u^* and q^* both increase with an increase in $P_{penalty}$ and y. However, u^* decreases while q^* increases with an increase in L and θ.

[Insert Table 1 about here]

Increases in the penalty rate or expected emissions both reflect an increase compliance costs, resulting in an increase in the demand for each compliance instrument. However, changes in the import limit and initial allowances, θ and L, cause a change in the relative cost of abatement verses offsets. Increasing the import limit, θ, induces a substitution of offset credits for allowances and simultaneously reduces abatement as offset credits become more plentiful in the market. Conversely, increasing L results in the opposite relative price effect as the capped party substitutes allowances for offsets and simultaneously decreases abatement. Also note that increasing allowances implicitly relaxes the import limit on offset credits, because the import limit is set as a proportion of initial allowances, thereby dampening the relative price effect of a change in initial allowances but further reinforcing reduced abatement.[6] To get more nuanced predictions from the model about how the policy parameters affect the price spread between emission allowances and offset credits, we must make assumptions about functional forms and parameter values in the model. In the next section we calibrate the model as best we can to the EU-ETS in order to explore the magnitude of the comparative statics we derived in this section.

3. Calibration

Each cost function is assumed to be quadratic and convex. The second term of equation (1), i.e. the cost of offset projects for the uncapped party, is defined by

(12) $$C_{offset}(q) = \frac{k}{2}q^2$$

where k is a scaling parameter and q is offset credits. The abatement cost function for the capped party, the first term of equation (4), has a similar functional form.

(13) $$TAC(u) = \frac{c}{2}u^2$$

where c is scaling parameter, and u is abatement. Specifying marginal abatement cost and marginal offset cost as linear is admittedly simplistic, but it is probably a reasonable compromise. Hintermann(2010) notes that abatement is typically accomplished by electric generating firms switching fuel from coal to natural gas. If input prices remain constant, then marginal abatement cost would be linear as the firms switch increasing input proportions from coal to natural gas.

[6] The European Commission sets the offset usage limits found in Article 11.a of Directive 2009/29/EC Amendment (European Commission). Regulated parties are allowed to use offsets up to a certain percentage of total EUA allocations during the Phase 2 (2008~2012). The amount of offsets allowed varies according to the types of regulated parties.

Further, this specification is convenient because it allows us to retain a tractable closed form solution.

Further, we must make distributional assumptions for the uncertain parameters: realized emissions, y, and offset credit default rate, ε. We assume both parameters follow a beta probability distribution.[7] We choose the beta distribution because it is flexible enough to model the nature of uncertainty in this market in a reasonable way and at the same time this distributional assumption is convenient because we are able to derive a closed form solution.[8] The parameter y represents baseline emissions that would occur with zero abatement. Therefore it is most commonly assumed to follow a symmetric probability distribution such as the normal; see Bushnell (2011). Following this convention we choose parameters of the beta that result in a symmetric distribution around the expected value of emissions, $f(y) \sim Beta(3,3,\bar{y})$, where $\bar{y}$ is the upper bound of uncertain baseline emissions[9].

Regarding the offset credit default factor, ε, we assume that the probability of default is decreasing in the default factor; i.e., the highest probability occurs at a zero default rate, and the lowest probability is assigned to 100% default. This follows the assumption of Huang (2007), based on a survey on the default risk of offset projects in China. Accordingly, we assume that the default factor is distributed according to the following beta p.d.f. to retain the aforementioned characteristics: $g(\varepsilon) \sim Beta(5,1)$.[10]

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[7] The beta probability density function for realized emissions is $f(y;\, \alpha,\, \beta, \bar{y}) = \frac{(\bar{y}-y/\bar{y})^{-1+\beta}(\frac{y}{\bar{y}})^{-1+\alpha}}{\int_0^1 (1-z)^{-1+\alpha} z^{-1+\beta} dz}$:where α and β are shape parameters and $\bar{y}$ is the parameter that scales realized emissions, y, to the interval [0,1]. The beta probability distribution for the default rate is $g(\varepsilon;\, \alpha, \beta) = \frac{(1-\varepsilon)^{-1+\beta}(\varepsilon)^{-1+\alpha}}{\int_0^1 (1-z)^{-1+\alpha} z^{-1+\beta} dz}$.

[8] The closed form solution is available upon request.

[9] There has been much literature about the approximation of probability density functions using the beta distribution (Peizer and Pratt, 1968; Alfers and Dinges, 1984; Kerman, 2011).

[10] See Appendix B to see the shapes of Beta distribution.

Table 2 contains the full set of parameter assumptions required to calibrate our model. The import limit, penalty rate and free allocation of allowances follow the levels of the Phase 2 (2008~2012) in the EU-ETS: 13.4%, 100 Euros, and 6 billion Tonnes respectively. While our model is stylized, our goal is to calibrate our model to the EU-ETS to the degree possible so we choose the marginal abatement cost, c, and the marginal offset project cost, k, such that the model's price trajectories of allowances and offsets pass through the average value of price expectations for Phase 3 (2013~2020) EUAs and CERs, €19.23 EUA (*GHG Market Sentiment Survey 2012,* IETA). The upper limit on total emissions, $\bar{y}$, is equal to 13 billion tons in each period so that $E[y_t]$ equals the estimates from the tenth report of Energy and Climate Change Committee in U.K. parliament (2012).

[Insert Table 2 about here]

In figure 2, we illustrate the equilibria of abatement and offset credit quantities, u^* and q^* respectively, as the import limit, θ, varies from 0 to 1. The dotted line that partitions the θ-Quantity plane represents the line, $q = L\theta$, which is the threshold between the two functional forms in equation (3).[11] The left hand side of the figure 2 represents the case where usable offset credits are defined by $Q = q\varepsilon$, where Q does not reach the import limit; whereas, the right hand side of the figure represents the case where usable offset credits are defined by a binding import limit, $Q = L\theta$.[12] The thick solid line shows that the optimal offset credit quantity increases as the import limit increases until the import limit is no longer binding. Increasing the import limit beyond the point where $q^* = L\theta$ does not increase q^* further. The thin solid line shows that optimal abatement u^* decreases as the import limit increases until the import limit is no longer

[11] $E\min\{q\varepsilon, L\theta\} = \int_0^{\frac{L\theta}{q}} q\varepsilon f(\varepsilon)d\varepsilon + \int_{\frac{L\theta}{q}}^{1} L\theta f(\varepsilon)d\varepsilon$ when $1 > \frac{L\theta}{q} > 0$

$E\min\{q\varepsilon, L\theta\} = \left(\frac{k}{k+1}\right)q$ when $1 < \frac{L\theta}{q}$

binding, and increasing the import limit beyond $q^* = L\theta$ does not produce further reductions in abatement.

One consequence of default risk in the offset market, is that occasionally over compliance will be observed in years where there was an unexpectedly large draw from the ε distribution. Investment in offset projects is based on the party's expectations about how many offsets will actually be delivered. If ε turns out to be larger than expected, the party will received more offsets than they anticipated and possibly even more than they need for compliance.

[Insert Figure 2 about here]

The quantity equilibria of abatement and offset credits shown in figure 2, imply price equilibria, which we derived from equations (10) and (11) and display in figure 3.

[Insert Figure 3 about here]

Given the baseline calibration, figure 3 shows a persistent spread between allowance and offset credit prices. We wish to explore the factors that explain the spread in more detail so we explicitly define the spread as

(14) $$Spread = P_{allowance} - P_{offset} =$$

$$MAC(u^*: c, k, P_{penalty}, \bar{y}, L, \theta, q^*) - MC(q^*: c, k, P_{penalty}, \bar{y}, L, \theta, u^*)$$

Here, the price equilibrium shows how two main factors, the import limit and the uncertainties in offset credits, can affect the spread between allowance and offset credit prices:

The import limit, θ, can be chosen from 0% to 100% of initial allowance endowments, and we illustrate how this impacts the spread between allowances and offset credit prices in figure 3. With a restrictive import limit (small θ) the allowance price is high because few carbon offsets can be imported and used for compliance instead of allowances. Correspondingly, the price of offset credits is low because few credits are demanded by the capped party. Therefore, the more restrictive the import limit, the higher the price spread between allowances and offset credits will be. Conversely, with a generous import limit the spread narrows to a certain point we define as the *convergence threshold*: the point at which the import limit is no longer binding in the offset credit market. In figure 3, this occurs at an import limit of about 60%. However, when the import limit is no longer binding a positive and constant spread between the two persists because there is delivery risk in the offset credit market, and the magnitude of this spread is determined entirely on the degree of risk in offset credit delivery, ε.

4. Analysis

In this section we show how the market equilibrium responds to changes in market conditions. First, we accomplish this with a sensitivity analysis performed on the parameters we calibrated in the previous section. Next, we will provide a graphical representation of the mechanisms driving this market equilibrium, with special emphasis on the spread between the emission allowance and offset credit prices.

4.1. Sensitivity Analysis

Table 3 shows the sensitivity of prices and the convergence threshold to a change of plus and minus 10% in each parameter. Supply side parameters include abatement costs and offset credit cost parameters; demand side parameters include noncompliance penalty and the emission cap. Note that the results show a marked asymmetry in the price responses of allowances and offset credits, respectively: a large response in the price of allowances is coupled by a small response in the price of offset credits and vice versa.

To support table 3, figure 4 graphs the price response when the parameters are perturbed as in table 3. We illustrate only examples of positive perturbation since the responses of -10% and +10% perturbations are roughly the same but opposite directions.

[Insert Table 3 about here]

[Insert Figure 4 about here]

In figure 4, the solid lines refer to the baseline case discussed above and the dashed lines represent the price trajectories after perturbing the parameters as indicated. In the 1st panel figure 4, we illustrate how increasing the abatement cost coefficient, *c*, moves the price of both credits in the same direction. Recall from table 3 that the sensitivity of $P_{allowance}$ to a 10% increase in abatement costs is much larger than that observed for P_{offset}: a 4.8% increase verses a 0.56% increase, respectively, which widens the price spread. An increase in *c* causes the capped party to substitute offset credits for allowances, making the import limit more binding and raising the convergence threshold by 2.61%. This is because expensive abatement induces the capped party to use more credits, meaning the prices of allowances, $P_{allowance}$, and the price of substitutable offset credits, P_{CER} would increase. Since abatement cost directly affects the allowance market and indirectly affects the offset credit market, we see a larger price response in allowances than offset credits, contributing to a wider spread between the two.

The similar effect is observed for an increase in offset project cost, *k*, in the 2nd panel figure 4, but the roles are reversed: recall a 0.05% increase in $P_{allowance}$ versus a 7.84% increase in P_{offset} in table 3, narrowing the price spread. An increase in offset project costs causes capped party to substitute allowances for offset credits, making the import limit less binding and reducing the convergence threshold by 6.43%.

The 3rd panel figure 4 illustrates the effect of an increase in the noncompliance penalty, $\boldsymbol{P_{penalty}}$: a 5.11% increase in $P_{allowance}$ versus a 0.77% increase in P_{offset}, widening the price spread. The increase in the relative price of allowances verses offset credits induces more demand for offset credits, causing the import limit to be more binding and increasing the convergence threshold by 3.23%.

Similar to the effect of an increase in $\boldsymbol{P_{penalty}}$, the 3rd panel of figure 4 can also be used to illustrate the effect of an increased probability of emissions. Table 3 shows a steeper increase in $P_{allowance}$ than P_{offset} in this case: a 12.52% increase in $P_{allowance}$ versus a 1.85% increase in P_{offset}, widening the price spread. The increase in the relative price of allowances verses offset credits induces more demand for offset credits, causing the import limit to be more binding and increasing the convergence threshold by 12.65%.

Finally, the regulator can change the initial supply of allowances and the 4th panel figure 4 illustrates this effect. Note that unlike the previous cases, the change in the initial endowment of allowances causes the price of allowances and offset credits to move in opposite directions. Table 3 shows a 10% increase in the supply of allowances lowers $P_{allowance}$ by 12.01% but raises P_{offset} by 6.37%, which narrowing the price spread. Also unlike the previous cases, we see that the import limit becomes less binding and the convergence threshold decreases by 20.31%. The reason for the unusual response relative to the other cases is that since the import limit is defined as the fraction of total allowances (as seen in equation 2), increasing the initial endowment of allowances effectively also implicitly makes the import limit more lenient. This also results in larger price effects in percentage terms than we saw in the previous cases.

Comparing the price response across the cases considered we observe some commonalities. We see asymmetric price responses to perturbations in the model's parameters; the stronger price effect is associated with the compliance instrument most directly affected. For example, increasing the cost of abatement makes compliance more difficult, thus raising the price of both compliance instruments. However, since the cost of abatement directly impacts the allowance market and indirectly affects the offset market the price effect in the allowance market is stronger and thus widens the price spread between the two compliance instruments. We elaborate on this intuition in the next section.

4.2. Conceptual Analysis

We provide a conceptual analysis of the allowance and emission markets to further explain the persistent spread between allowance and offset prices observed in the previous section.

[Insert Figure 5 about here]

Panel 1 in figure 5 depicts the offset market. In this figure $MB(q|u_1)$ is the demand for offsets when there is no delivery risk and no import limit. The marginal cost of producing offset credits is denoted by $MC(q)$. Panel 2 depicts the allowance market with the demand for allowances denoted by $MB(u|q)$ and the supply denoted by $MAC(u)$. We see that the baseline equilibrium (q_1, u_1) generated by the case where there is no delivery limit or delivery risk has allowances and offsets equal in price. The dotted curve, $MB(\varepsilon \cdot q|u_1)$, represents the offset credit demand curve when there are both delivery risks and an import limit. Delivery risk is apparent in the figure because to the left of the import limit, the dotted line is equal to the baseline demand curve for offsets discounted by ε. The import limit causes the steep decline in the offset demand curve as the import limit is a demand restriction. The possibility of default of some offset credits creates the possibility for demand off offset credits beyond the import limit. The import limit and delivery risk lowers the equilibrium to q_2, a reduction in both the equilibrium quantity and price. This effect has consequences in the allowance market; a reduction in the use of offset credits causes the demand for allowances, a substitute, to increase. This is depicted in the rightmost graph of panel 1 as a shift to the right in $MB(u|q_2)$. The allowance market must accommodate the excess demand produced by the wedge in the offset credit market. This increases the amount of abatement and the price of allowances (represented by the allowance market equilibrium moving from u_1 to u_2). With the probability of default and the import limit imposed, the equilibria moves from $\{q_1, u_1\}$ to $\{q_2, u_2\}$ and a spread between the allowance and offset credit prices is observed.

Panel 2 shows the case where the import limit is relaxed enough so that it does not affect the offset credit equilibrium. A lenient import limit results in offset credit demand according to $MB(u|q_3)$, which increases the offset credit price and quantity equilibrium ($q_2 \rightarrow q_3$). This new equilibrium in the offset credit market decreases the marginal benefit of abatement, resulting in a lower price and quantity equilibrium for allowances ($u_2 \rightarrow u_3$) and reducing the spread between allowances and offset credits. However, a spread between allowances and offset credits remains due the continued presence of delivery risk in the offset credit market.

Furthermore, figure 6 illustrates the comparative statics, in section 2.3, and sensitivity analysis in Section 4.1. In particular, figure 6 illustrates how the asymmetric price changes shown in section 4.1 come about.

[Insert Figure 6 about here]

In the 1st panel of figure 6, we illustrate the effect of an increase in marginal abatement cost. An increase in c shifts MAC(u) to the left and initially increases $P_{allowance}$ ($u_1 \rightarrow u_2$) and lowers the equilibrium amount of abatement. Now since u is a shifter of offset credit demand (see equation 9), decreased abatement increases demand for offset credits $\left(MB(q|u_1) \rightarrow MB(q|u_2)\right)$, which increases the offset credit price. Thus, $P_{allowance}$ increases more than P_{offset}.

In the 2nd panel, we illustrate the effect of an increase in the marginal cost of an offset credit project. This increases P_{CER} ($q_1 \rightarrow q_2$) and lowers the amount of offset credits, q. Since q is a substitute for allowences, the demand for allowances increases, which increases $P_{allowance}$ ($u_1 \rightarrow u_2$). Thus, P_{offset} increases more than $P_{allowance}$.

The 3rd panel explains the effect of an increase in the penalty rate, $P_{penalty}$. An increase in the penalty rates increases the opportunity costs of noncompliance, which shifts the demand of both allowances and offset credits ($u_1 \rightarrow u_2$; $q_1 \rightarrow q_2$) to the right. However, this demand side effect is limited by the import limit in the offset market, whereas, this does not exist in the allowance market. The 3rd panel also can explain the effect of increased emissions since an increase in emissions also increases both the demand curves for allowances and offsets ($u_1 \rightarrow u_2$; $q_1 \rightarrow q_2$). Again, the price increase in the offset credit market is limited by the import limit compared to the allowance market.

In the 4th panel, an increase in the supply of allowances initially decreases both allowance and offset credit prices ($u_1 \rightarrow u_2$; $q_1 \rightarrow q_2$), but the increased supply of allowances also increases the import limit. Therefore, P_{offset} actually rises ($q_2 \rightarrow q_3$) when the import limit is binding.

5. Conclusion

Many countries have introduced tradable emission allowances and offset credits as part of a regional, national, or multi-national plan for regulating greenhouse gas emissions and compliance with international commitments. Also, many kinds of carbon offset schemes have been developed to complement the emission allowance markets.

Our model reflects stylized features of an emission allowance and the carbon offset market. We use a partial equilibrium model to highlight the asymmetric price response of the emission allowance market and the carbon offset market to changing market parameters. Because emission allowances and carbon offsets are substitutes, the direct effect of a parameter change in one market is always complemented by an indirect effect in the other market. Thus we can generate a large price response in the market directly impacted and a smaller price response in the other market, impacting the spread between the two. Further, the indirect price effect can be counterintuitive in sign. For example, if the regulator decreases the supply of allowances, which has long been considered as the most effective market booster, this will naturally increase the price of allowances. However, ceteris peribus this will decrease the price of carbon offsets since the import limit is set as a proportion of the supply of allowances. We showed how a change in abatement costs, offset project costs, the compliance penalty, and emissions cause allowance and offset credit prices to move in the same direction while a change in the import limit and emission cap causes allowance and offset credit prices to move in opposite directions.

Our analyses give some intuition about the linkage between allowance and offset markets and their interaction with policy. For example, some have argued that there cannot be concurrent incentive for both investment in offset projects in developing countries and cost reducing innovations in abatement technologies in Annex 1 countries. The argument is that if offset projects are profitable, then there will be little incentive to invest in research to decrease abatement costs. Our results suggest that the effect would be minimal as long as the import limit on offset credits is binding. Although a decrease in abatement costs would decrease both allowance and offset prices, our model predicts that the brunt of the effect would be felt in the allowance market. That is, a decrease in abatement cost significantly reduces allowance prices but only minimally reduces offset prices, narrowing the spread and having a minimal impact on the profitability of offset credit projects in developing countries. Therefore, our model predicts that innovation in abatement technologies will not have a large adverse effect on offset projects.

We see several possible extensions of our model. In this paper, we assume both the capped and uncapped parties have identical information about the default risk. In reality there are likely to be significant informational asymmetries in the offset markets, especially with regard to the default risk. The literature would benefit from a thorough analysis of the implications of the informational asymmetries in this market. Also, we do not consider inter-temporal carbon credit transfers because we wanted to to focus on the price determination within a single compliance period. A model that allows inter-temporal credit transfers would mitigate the effects of unexpected emission or default realizations, and therefore result in less volatile price predictions. Additionally, the policy levers available to the regulator would become less effective because market participants can lessen these shocks by banking or borrowing credits.

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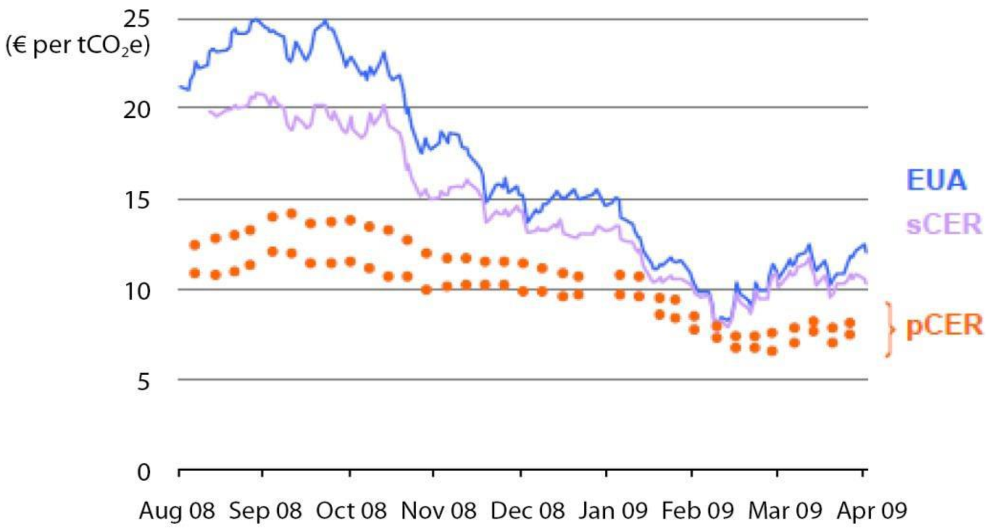
Figure 1.



Source: World Bank, 2009 (Spot EUA and sCER: Bluenext; average primary CER price for categories b and c: IDEA Carbon)[13]

[13] The primary CER is categorized in three groups: "Category A": negotiating terms, "Category B": in exclusive negotiations, "Category C": contracted. Category A is usually not cited as the primary CER index because it is in too early stages to be informative figures for delivery projections. (http://www.tradingemissionsplc.com/Report2008/investment_advisers.swf).

Figure 2 : Optimal equilibrium of abatement and offset credits

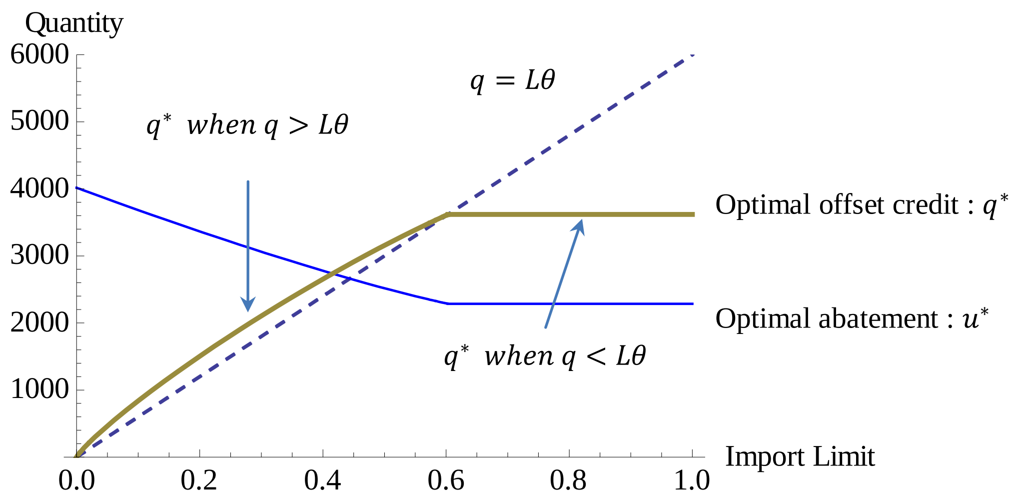


Figure 3 : Price equilibrium trajectory when the offset credit import limit changes.

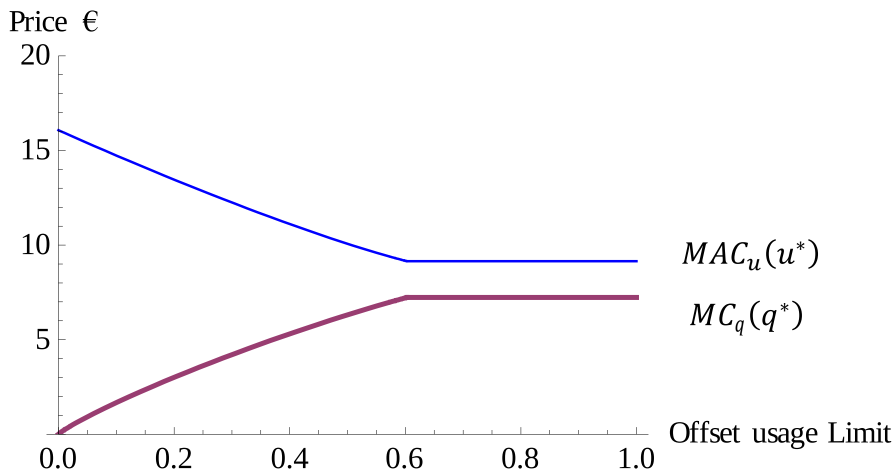
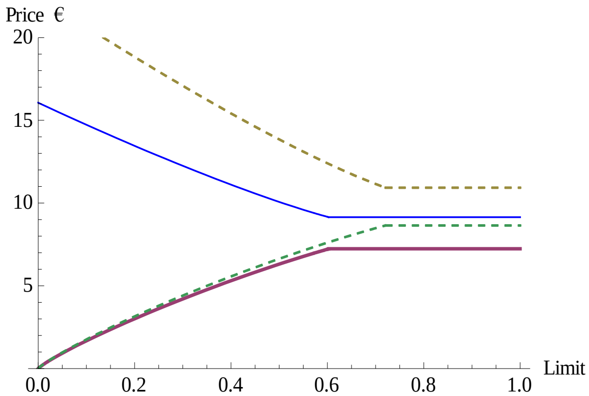
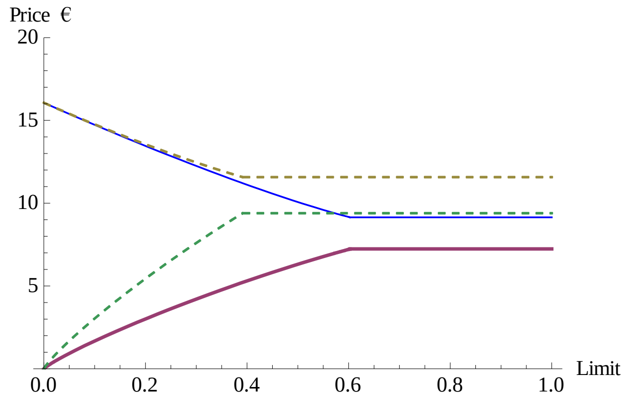


Figure 4: Price equilibrium trajectory when the market conditions change

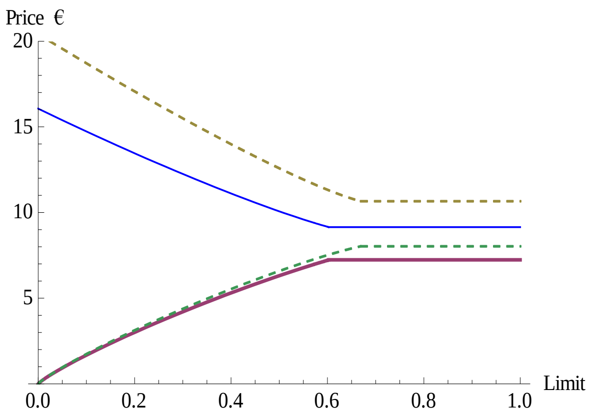
Panel 1) Effect of abatement cost shock



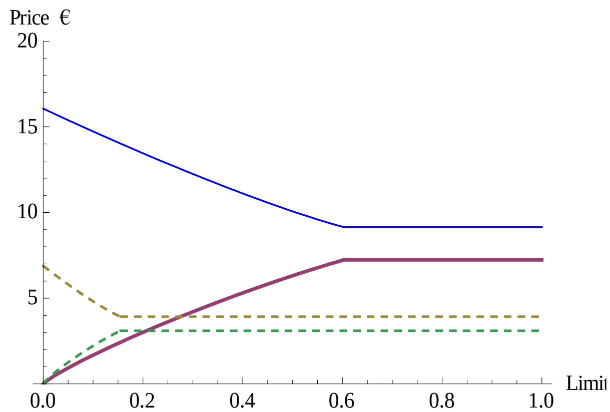
Panel 2) Effect of offset project cost shock



Panel 3) Effect of penalty change or uncertainty in emissions



Panel 4) Effect of free endowment of allowances

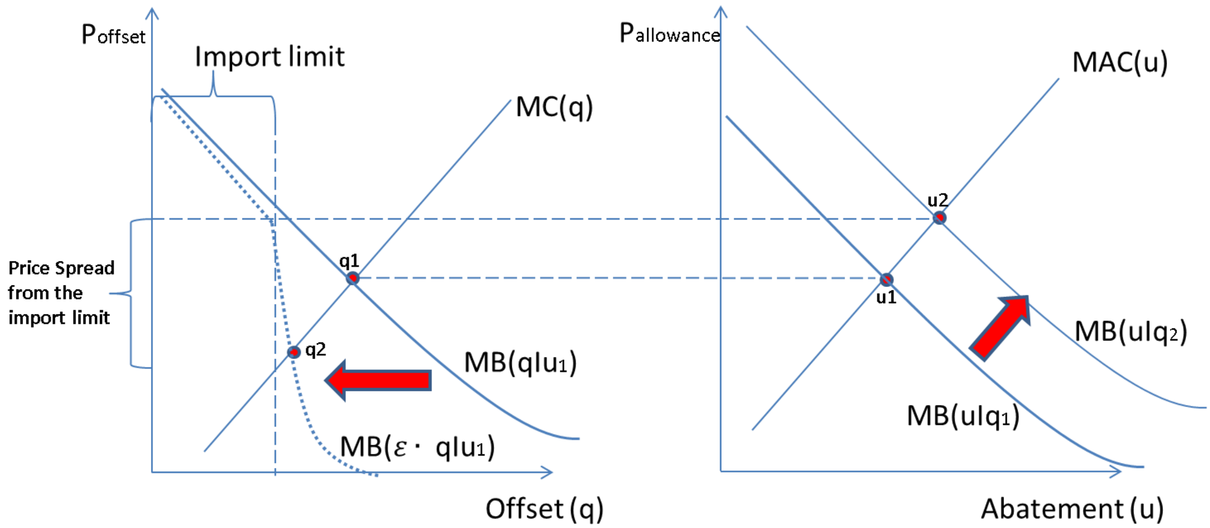


* Dotted line (after the change), solid line (before the change)

** Note that all the graphics in figure 4 reflex the effect of more than a 50% change in each parameter to show the effect more vividly.

Figure 5. Persistent Price Spread

Panel 1) Effect of the import limit on the spread



Panel 2) Effect of the partial equilibrium adjustment

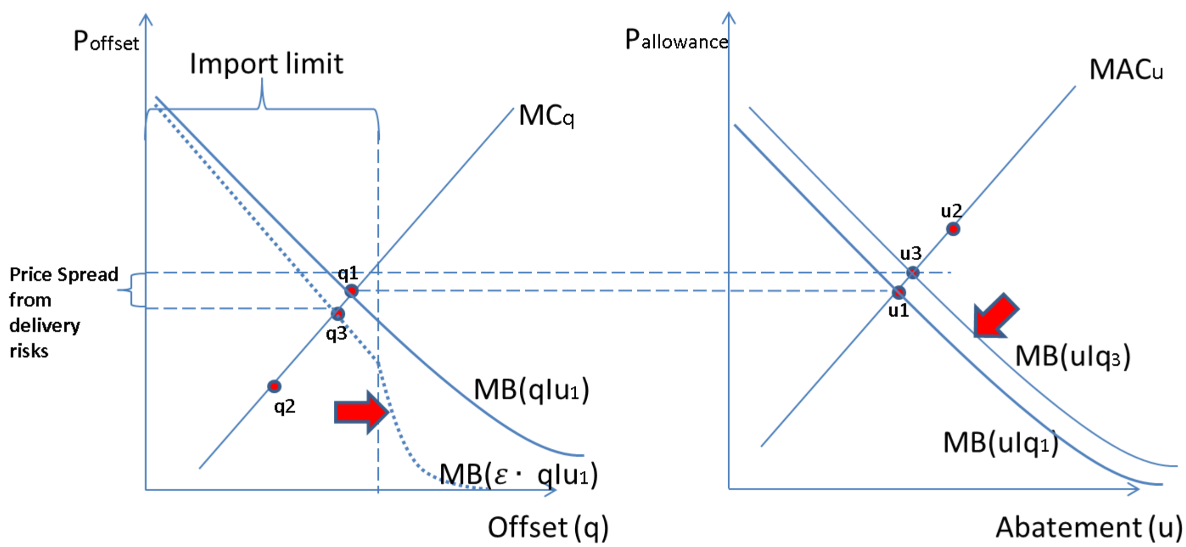
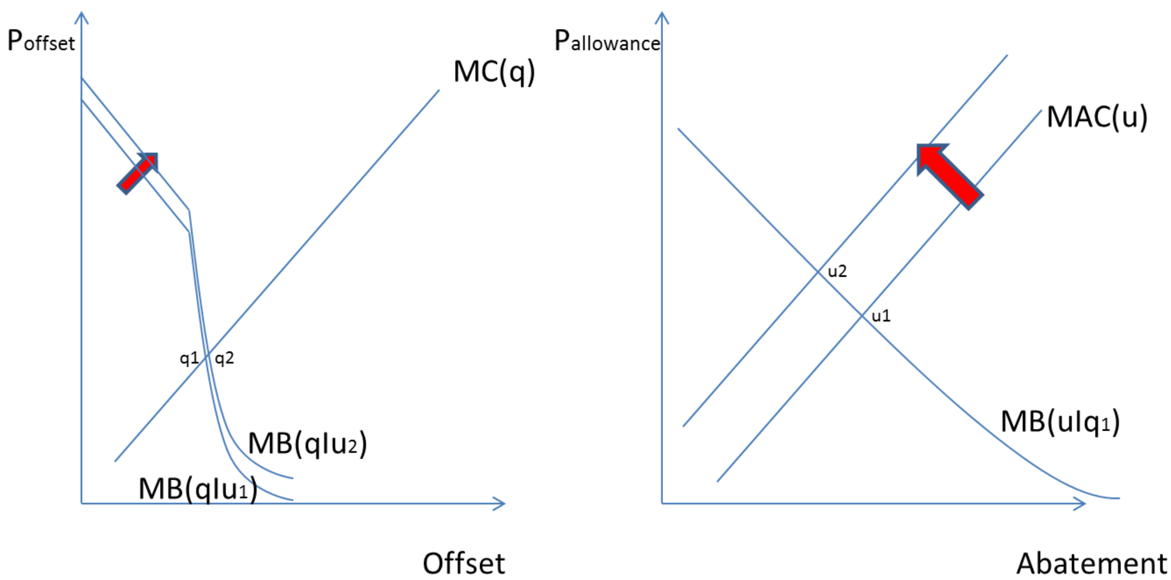
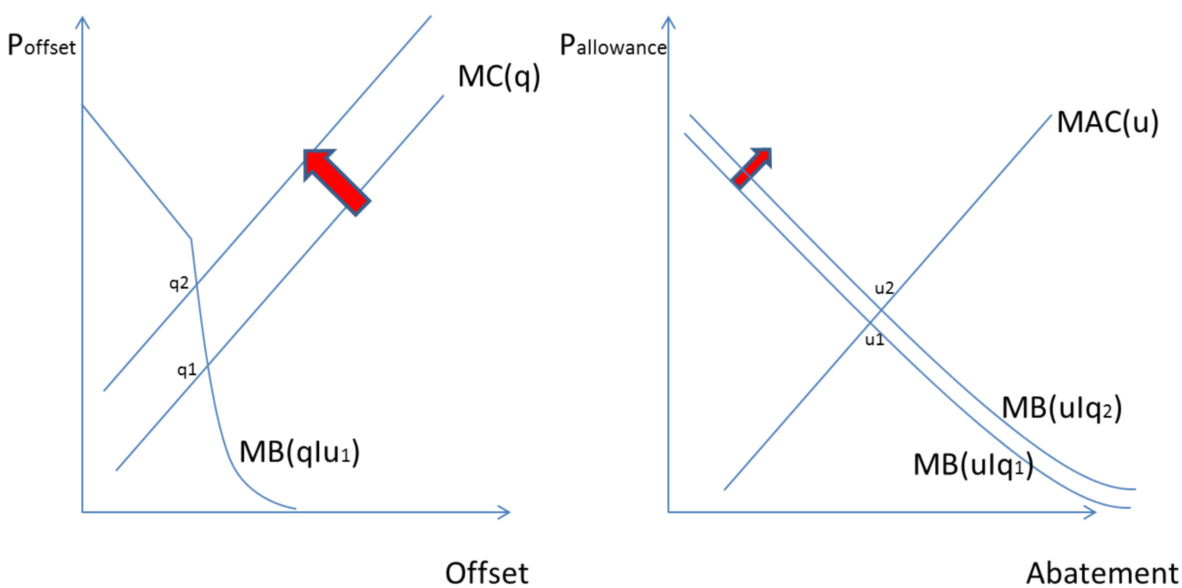


Figure 6: Asymmetric Market Interaction

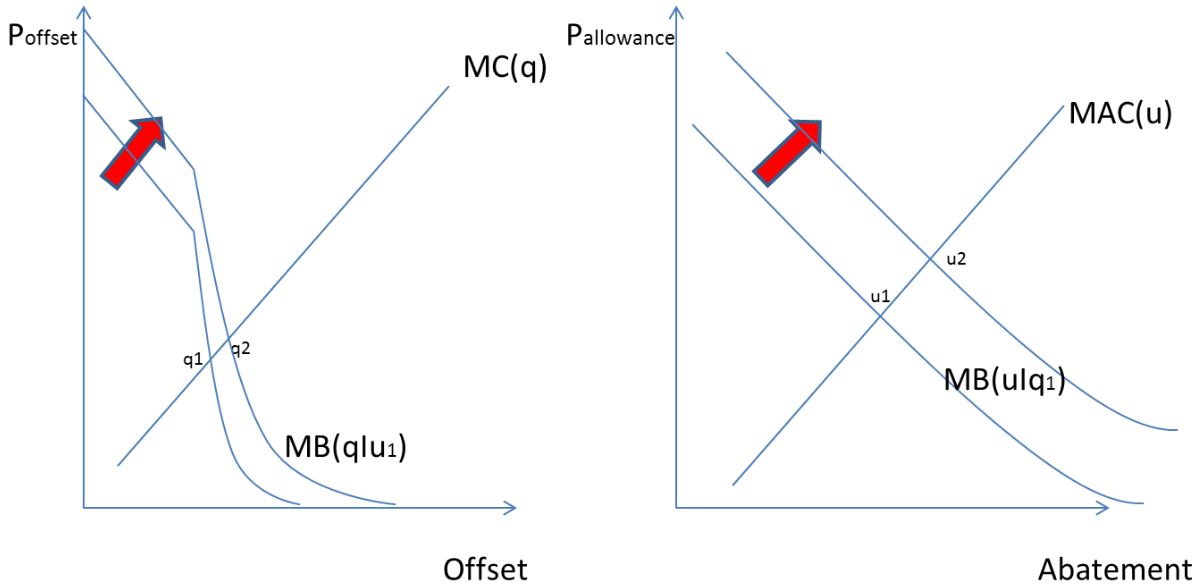
Panel 1) Effect of abatement cost shock (correspond to the panel 1 of figure 4)



Panel 2) Effect of offset project cost shock (correspond to the panel 2 of figure 4)



Panel 3) Effect of penalty change or uncertainty in emissions (correspond to the panel 3 of figure 4)



Panel 4) Effect of free endowment of allowances (correspond to the panel 4 of figure 4)

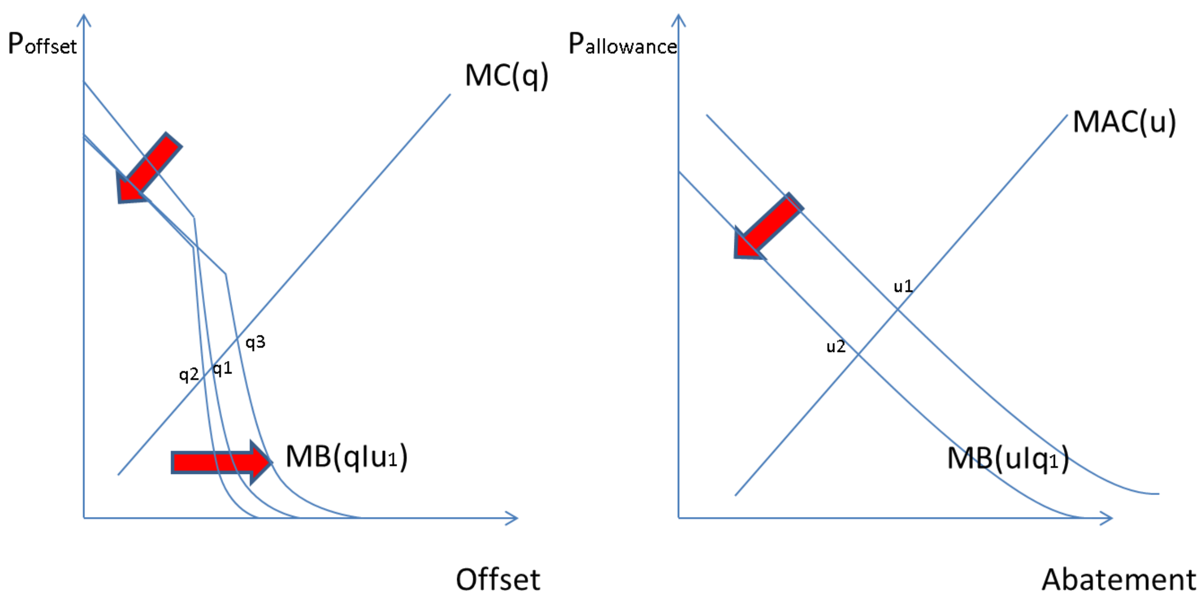


Table 1: Comparative Static Results

Changes in parameters	Changes in equilibrium	
$P_{penalty} \uparrow$,	$u^* \uparrow$	$q^* \uparrow$
$y \uparrow$,	$u^* \uparrow$	$q^* \uparrow$
$\theta \uparrow$,	$u^* \downarrow$	$q^* \uparrow$
$L \uparrow$	$u^* \downarrow$	$q^* \uparrow$

Note: a change in both the penalty rate and expected emissions causes a change in the same direction in the demand for each compliance instrument. However, a change in the import limit and initial allowances result in an opposite change in abatements and offsets.

Table 2: Parameter Assumptions

	Parameters	Values*
Supply side	Cost coefficient for Abatement, c	0.004
	Cost coefficient for offset project, k	0.002
Demand side	Penalty rate, $P_{penalty}$	70
	Uncertain emission upper limit, $\bar{y}$	13,000
	Free endowment of allowances, L	6,000

* Parameters are chosen from the stylized fact of European carbon market. We choose the cost coefficients that roughly illustrate current spot EUA / CER prices at the current import limit (13.4% of total EUA). The upper limit of emissions is assumed to let its average be slightly higher than the amount of EUA endowment (6 billion Tonne of CO_2 for a compliance period) to encourage abatement.

Table 3: Sensitivity Analysis (% change)

	Parameters		$P_{allownace}$	P_{offset}	threshold
Supply side	Cost coefficient for Abatement, c	+10%	+4.80 %	+0.56 %	+2.61 %
		-10%	-4.59 %	-0.66 %	-3.09 %
	Cost coefficient for offset project, k	+10%	+0.05 %	+7.84 %	-6.43 %
		-10%	-0.05 %	-8.63 %	+6.87 %
Demand side	Penalty rate, $P_{penalty}$	+10%	+4.80 %	+0.77 %	+3.23 %
		-10%	-5.88 %	-0.88 %	-3.81 %
	Uncertain emission upper limit, $\bar{y}$	+10%	+12.52 %	+1.85 %	+12.65 %
		-10%	-16.00 %	-2.40 %	-15.57 %
	Free endowment of allowances, L	+10%	-12.01 %	+6.37 %	-20.31 %
		-10%	+10.76 %	-7.32 %	+20.63 %

Appendix A: Comparative Statics

From the first order conditions in equation (8) and (9), the interior solution (u^*, q^*) is defined as below:

(A-1) $$\frac{\partial TAC(u)}{\partial u} + \frac{\partial P_{penalty} \cdot E\ max[y - L - u - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u} = 0$$

(A-2) $$\frac{\partial C(q)}{\partial q} + \frac{\partial P_{penalty} \cdot E\ max[y - L - u - \min[q\varepsilon, L\theta],\ \ 0]}{\partial q} = 0$$

The equation (A-1) implicitly defines $u^*(P_{penalty}|\bar{\Omega}), u^*(y|\bar{\Omega}), u^*(L|\bar{\Omega}), u^*(\theta|\bar{\Omega})$, and the equation (A-2) implicitly defines $q^*(P_{penalty}|\bar{\Omega}), q^*(y|\bar{\Omega}), q^*(\theta|\bar{\Omega}), q^*(L|\bar{\Omega})$, where the symbol Ω refers that other parameters are status quo in equation.

Regarding the effect of $P_{penalty}$, redefine (A-1) and (A-2) as

(A-3) $$\frac{\partial TAC\left(u^*(P_{penalty}|\bar{\Omega})\right)}{\partial u^*(P_{penalty}|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\ max\left[y - L - u^*(P_{penalty}|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(P_{penalty}|\bar{\Omega})} = 0$$

(A-4) $$\frac{\partial C\left(q^*(P_{penalty}|\bar{\Omega})\right)}{\partial q^*(P_{penalty}|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\ max\left[y - L - u - \min\left[q^*(P_{penalty}|\bar{\Omega})\varepsilon, L\theta\right],\ \ 0\right]}{\partial q^*(P_{penalty}|\bar{\Omega})} = 0$$

To see how $P_{penalty}$ affects the equilibria, we totally differentiate equation (A-3) and (A-4),

$$\left\{\frac{\partial^2 TAC\left(u^*(P_{penalty}|\bar{\Omega})\right)}{\partial u^*(P_{penalty}|\Omega)^2} + \frac{\partial^2 P_{penalty} \cdot E\ max\left[y - L - u^*(P_{penalty}|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(P_{penalty}|\bar{\Omega})^2}\right\} du^*(P_{penalty}|\bar{\Omega}) +$$

$$\left\{\frac{\partial E\ max\left[y - L - u^*(P_{penalty}|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(P_{penalty}|\bar{\Omega})}\right\} dP_{penalty} = 0$$

$$\Rightarrow \frac{du^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}} = -\frac{\left\{\frac{\partial\, E\, max\left[y-L-u^*\left(P_{penalty}|\bar{\Omega}\right)-\min[q\varepsilon,L\theta],\ 0\right]}{\partial u^*\left(P_{penalty}|\bar{\Omega}\right)}\right\}}{\left\{\frac{\partial^2 TAC\left(u^*\left(P_{penalty}|\bar{\Omega}\right)\right)}{\partial u^*\left(P_{penalty}|\bar{\Omega}\right)^2}+\frac{\partial^2 P_{penalty}\cdot E\, max\left[y-L-u^*\left(P_{penalty}|\bar{\Omega}\right)-\min[q\varepsilon,L\theta],\ 0\right]}{\partial u^*\left(P_{penalty}|\bar{\Omega}\right)^2}\right\}}$$

And,

$$\left\{\frac{\partial^2 C\left(q^*(P_{penalty}|\bar{\Omega})\right)}{\partial q^*(P_{penalty}|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty}\cdot E\, max[y-L-u-\min[q^*(P_{penalty}|\bar{\Omega})\varepsilon,L\theta],\ 0]}{\partial q^*(P_{penalty}|\bar{\Omega})^2}\right\} dq^*(P_{penalty}|\bar{\Omega}) +$$

$$\left\{\frac{\partial E\, max[y-L-u-\min[q\varepsilon,L\theta],\ 0]}{\partial q^*(P_{penalty}|\bar{\Omega})}\right\} dP_{penalty} = 0$$

$$\Rightarrow \frac{dq^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}} = -\frac{\left\{\frac{\partial\, E\, max\left[y-L-u-\min\left[q^*\left(P_{penalty}|\bar{\Omega}\right)\varepsilon,L\theta\right],\ 0\right]}{\partial q^*\left(P_{penalty}|\bar{\Omega}\right)}\right\}}{\left\{\frac{\partial^2 C\left(q^*\left(P_{penalty}|\bar{\Omega}\right)\right)}{\partial q^*\left(P_{penalty}|\bar{\Omega}\right)^2}+\frac{\partial^2 P_{penalty}\cdot E\, max\left[y-L-u-\min\left[q^*\left(P_{penalty}|\bar{\Omega}\right)\varepsilon,L\theta\right],\ 0\right]}{\partial q^*\left(P_{penalty}|\bar{\Omega}\right)^2}\right\}}$$

Since the objective function is a convex function with respect to (u, q) as a cost-minimizing objective function, the denominators of $\frac{du^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}}$ and $\frac{dq^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}}$ are positive; whereas, the nominators are negative.

Thus, we can conclude that $\frac{du^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}} > 0$ and $\frac{dq^*(P_{penalty}|\bar{\Omega})}{dP_{penalty}} > 0$

Regarding the effect of y, redefine (A-1) and (A-2) as

(A-5) $$\frac{\partial TAC\left(u^*(y|\bar{\Omega})\right)}{\partial u^*(y|\bar{\Omega})} + \frac{\partial P_{penalty}\cdot E\, max[y-L-u^*(y|\bar{\Omega})-\min[q\varepsilon,L\theta],\ 0]}{\partial u^*(y|\bar{\Omega})} = 0$$

(A-6) $$\frac{\partial C\left(q^*(y|\bar{\Omega})\right)}{\partial q^*(y|\bar{\Omega})} + \frac{\partial P_{penalty}\cdot E\, max[y-L-u-\min[q^*(y|\bar{\Omega})\varepsilon,L\theta],\ 0]}{\partial q^*(y|\bar{\Omega})} = 0$$

To see how y affects the equilibria, we totally differentiate equation (A-5) and (A-6).

$$\left\{\frac{\partial^2 TAC\left(u^*(y|\bar{\Omega})\right)}{\partial u^*(y|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max\left[y-L-u^*(y|\bar{\Omega})-\min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(y|\bar{\Omega})^2}\right\} du^*(y|\bar{\Omega}) +$$

$$\left\{\frac{\partial^2 P_{penalty} \cdot E\ max\left[y-L-u^*(y|\bar{\Omega})-\min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(y|\bar{\Omega})\partial y}\right\} dy = 0$$

$$\Rightarrow \frac{du^*(y|\bar{\Omega})}{dy} = -\frac{\left\{\frac{\partial^2 P_{penalty} \cdot E\ max[y-L-u^*(y|\bar{\Omega})-\min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(y|\bar{\Omega})\partial y}\right\}}{\left\{\frac{\partial^2 TAC\left(u^*(y|\bar{\Omega})\right)}{\partial u^*(y|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y-L-u^*(y|\bar{\Omega})-\min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(y|\bar{\Omega})^2}\right\}}$$

And,

$$\left\{\frac{\partial^2 C\left(q^*(y|\bar{\Omega})\right)}{\partial q^*(y|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max\left[y-L-u-\min[q^*(y|\bar{\Omega})\varepsilon, L\theta],\ \ 0\right]}{\partial q^*(y|\bar{\Omega})^2}\right\} dq^*(y|\bar{\Omega}) +$$

$$\left\{\frac{\partial^2 P_{penalty} \cdot E\ max\left[y-L-u-\min\left[q^*(y|\bar{\Omega})\varepsilon, L\theta\right],\ \ 0\right]}{\partial q^*(y|\bar{\Omega})\partial y}\right\} dy = 0$$

$$\Rightarrow \frac{dq^*(y|\bar{\Omega})}{dy} = -\frac{\left\{\frac{\partial^2\ P_{penalty} \cdot E\ max[y-L-u-\min[q^*(y|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(y|\bar{\Omega})\partial y}\right\}}{\left\{\frac{\partial^2 C\left(q^*(y|\bar{\Omega})\right)}{\partial q^*(y|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y-L-u-\min[q^*(y|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^2}\right\}}$$

Since the objective function is a convex function with respect to (u, q) as a cost-minimizing objective function, the denominators of $\frac{du^*(y|\bar{\Omega})}{dy}$ and $\frac{dq^*(y|\bar{\Omega})}{dy}$ are positive; whereas, the nominators are negative.

Thus, we can conclude that $\frac{du^*(y|\bar{\Omega})}{dy} > 0$ and $\frac{dq^*(y|\bar{\Omega})}{dy} > 0$

Regarding the effect of L, redefine (A-1) and (A-2) as

(A-7) $$\frac{\partial TAC\left(u^*(L|\bar{\Omega})\right)}{\partial u^*(L|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\ max[y - L - u^*(L|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(L|\bar{\Omega})} = 0$$

(A-8) $$\frac{\partial C\left(q^*(L|\bar{\Omega})\right)}{\partial q^*(L|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\ max[y - L - u - \min[q^*(L|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(L|\bar{\Omega})} = 0$$

To see how L affects the equilibria, we totally differentiate equation (A-7) and (A-8).

$$\left\{\frac{\partial^2 TAC\left(u^*(L|\bar{\Omega})\right)}{\partial u^*(L|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u^*(L|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(L|\bar{\Omega})^2}\right\} du^*(L|\bar{\Omega}) +$$

$$\left\{\frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u^*(L|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(L|\bar{\Omega})\partial L}\right\} dL = 0$$

$$\Rightarrow \frac{du^*(L|\bar{\Omega})}{dL} = -\frac{\left\{\frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u^*(L|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u \partial L}\right\}}{\left\{\frac{\partial^2 TAC\left(u^*(L|\bar{\Omega})\right)}{\partial u^*(L|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u^*(L|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(L|\bar{\Omega})^2}\right\}}$$

And,

$$\left\{\frac{\partial^2 C\left(q^*(L|\bar{\Omega})\right)}{\partial q^*(L|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u - \min[q^*(L|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(L|\bar{\Omega})^2}\right\} dq^*(L|\bar{\Omega}) +$$

$$\left\{\frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u - \min[q^*(L|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(L|\bar{\Omega})\partial y}\right\} dL = 0$$

$$\Rightarrow \frac{dq^*(L|\bar{\Omega})}{dL} = -\frac{\left\{\frac{\partial^2\ P_{penalty} \cdot E\ max[y - L - u - \min[q^*(L|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(L|\bar{\Omega})\partial y}\right\}}{\left\{\frac{\partial^2 C\left(q^*(L|\bar{\Omega})\right)}{\partial q^*(L|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\ max[y - L - u - \min[q^*(L|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(L|\bar{\Omega})^2}\right\}}$$

Since the objective function is a convex function with respect to (u, q) as a cost-minimizing objective function, the denominators of $\frac{du^*(L|\bar{\Omega})}{dL}$ and $\frac{dq^*(L|\bar{\Omega})}{dL}$ are positive. Contrary to previous cases, the nominator of $\frac{du^*(L|\bar{\Omega})}{dL}$ is positive, whereas, the nominator of $\frac{dq^*(L|\bar{\Omega})}{dL}$ is negative. Thus, we can derive different directions that $\frac{du^*(L|\bar{\Omega})}{dL} < 0$ and $\frac{dq^*(L|\bar{\Omega})}{dL} > 0$.

Regarding the effect of θ, redefine (A-1) and (A-2) as

(A-9) $$\frac{\partial TAC\left(u^*(\theta|\bar{\Omega})\right)}{\partial u^*(\theta|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\, max[y - L - u^*(\theta|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(\theta|\bar{\Omega})} = 0$$

(A-10) $$\frac{\partial C\left(q^*(\theta|\bar{\Omega})\right)}{\partial q^*(\theta|\bar{\Omega})} + \frac{\partial P_{penalty} \cdot E\, max[y - L - u - \min[q^*(\theta|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(\theta|\bar{\Omega})} = 0$$

To see how θ affects the equilibria, we totally differentiate equation (A-9) and (A-10),

$$\left\{\frac{\partial^2 TAC(u)}{\partial u^*(\theta|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\, max\left[y - L - u^*(\theta|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(\theta|\bar{\Omega})^2}\right\} du^*(\theta|\bar{\Omega}) +$$

$$\left\{\frac{\partial^2 P_{penalty} \cdot E\, max\left[y - L - u^*(\theta|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0\right]}{\partial u^*(\theta|\bar{\Omega})\partial\theta}\right\} d\theta = 0$$

$$\Rightarrow\ \frac{du^*(\theta|\bar{\Omega})}{d\theta} = -\frac{\left\{\frac{\partial^2 P_{penalty} \cdot E\, max[y - L - u^*(\theta|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(\theta|\bar{\Omega})\partial\theta}\right\}}{\left\{\frac{\partial^2 TAC\left(u^*(\theta|\bar{\Omega})\right)}{\partial u^*(\theta|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\, max[y - L - u^*(\theta|\bar{\Omega}) - \min[q\varepsilon, L\theta],\ \ 0]}{\partial u^*(\theta|\bar{\Omega})^2}\right\}}$$

And,

$$\left\{\frac{\partial^2 C\left(q^*(\theta|\bar{\Omega})\right)}{\partial q^*(\theta|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\, max\left[y - L - u - \min\left[q^*(\theta|\bar{\Omega})\varepsilon, L\theta\right],\ \ 0\right]}{\partial q^*(\theta|\bar{\Omega})^2}\right\} dq^*(\theta|\bar{\Omega}) +$$

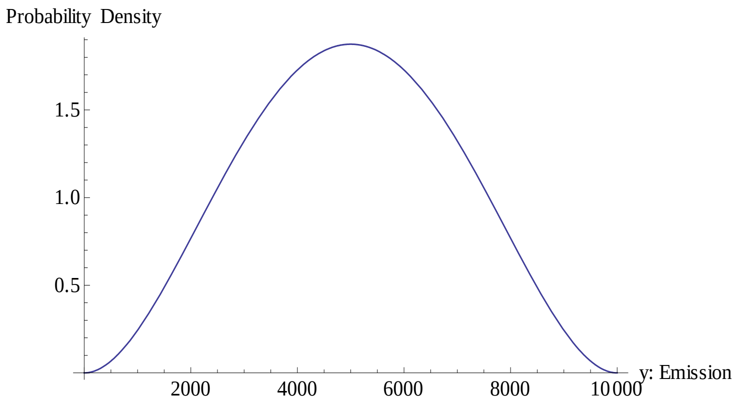
$$\left\{\frac{\partial^2 P_{penalty} \cdot E\, max\left[y - L - u - \min\left[q^*(\theta|\bar{\Omega})\varepsilon, L\theta\right],\ \ 0\right]}{\partial q^*(\theta|\bar{\Omega})\partial\theta}\right\} d\theta = 0$$

$$\Rightarrow\ \frac{dq^*(\theta|\bar{\Omega})}{d\theta} = -\frac{\left\{\frac{\partial^2\ P_{penalty} \cdot E\, max[y - L - u - \min[q^*(\theta|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(\theta|\bar{\Omega})\partial\theta}\right\}}{\left\{\frac{\partial^2 C\left(q^*(\theta|\bar{\Omega})\right)}{\partial q^*(\theta|\bar{\Omega})^2} + \frac{\partial^2 P_{penalty} \cdot E\, max[y - L - u - \min[q^*(\theta|\bar{\Omega})\varepsilon, L\theta],\ \ 0]}{\partial q^*(\theta|\bar{\Omega})^2}\right\}}$$

Since the objective function is a convex function with respect to (u, q) as a cost-minimizing objective function, the denominators of $\frac{du^{*}(\theta|\bar{\Omega})}{d\theta}$ and $\frac{dq^{*}(\theta|\bar{\Omega})}{d\theta}$ are positive. Contrary to previous cases, the nominator of $\frac{du^{*}(\theta|\bar{\Omega})}{d\theta}$ is positive, whereas, the nominator of $\frac{dq^{*}(\theta|\bar{\Omega})}{d\theta}$ is negative. Thus, we can derive different directions that $\frac{du^{*}(\theta|\bar{\Omega})}{d\theta} < 0$ and $\frac{dq^{*}(\theta|\bar{\Omega})}{d\theta} > 0$.

Appendix B: Beta Probability Density Function

Probability Density function of baseline emissions (y)



Probability Density function of the CER default factor (ε)